



1. Let S be the set of all integers of the form $P^2 - 1$ where P is a prime number greater than 5. Let N be the largest integer that divides every member of S . Find, with proof, the value of N .
2. In triangle ABC , $(\cos B)(\cos C) = \cos A$. Find, with proof, the numerical value of $(\tan B)(\tan C)$.
3. Suppose that $n + 1$ boys are lined up shoulder-to-shoulder from left to right in a straight line. Prove that it is always possible to select $n + 1$ boys to take one step forward so that, going from left to right, their heights are either consistently increasing or consistently decreasing.
4. The lengths of the sides of triangle ABC are in the ratio of 4:5:6. The bisector of the largest angle of the triangle is drawn. Prove that one of the two triangles formed also has sides whose lengths are in the ratio of 4:5:6.
5. All the factors of the polynomial $P(x) = x^3 + ax^2 + bx + c$ are linear with integer coefficients, and neither a nor b are zero or one. Find all possible pairs (a, b) , and prove that you have found them all.

3 3 ± 3 6LQFH 3 LV D SULPH DQG 3 ! 3 LV RG
DQG 3 PXVW EH FRQVHFXWLYH HYHQ QXPEHUV 7K
PXOWLSOH RI3 ZKLV KYRWDQOH E\ 3\$DQB 3VLQFHUH ±
WKUHH FRQVHFXWLYH LQWHJHUV RQH RI WKHP PXVW
HLWKHU ± PXVW EH D PXOWLSOHGLYLVL7KHU HEIRUH
,I 33 ,I 3 3 6LQFH LV WKH JFG RI DQ
N=24 ODUJHVW

RV% FRV& %FRFRV&± V%QVLQ&

± FRV% FRV& VLQ% VLQ&

7KHUHIRUH FRV% FRV& VLQ% VLQ& VR WKDW

$$\frac{VLQ}{FRV}$$
 WDI

\$VVXPH LW LV LPSRVVLEOH WR ILQG Q ER\ V LQ DVF
:H ZLOO VKRZ WKDW LW LV WKHQ SRVVLEOH WR ILQG Q
6WDUWLQJ ZLWK WKH ILUVW ER\ ZH IRUP D 'FOXE' LQ V
ER\ WDOOHU WKDQ KLP WKH ILUVW ER\ WDOOHU WKDQ
WKDQ Q ER\ V LQ WKLV FOXE 1H[W ZH FUHDWH D VHFRQ
WKH ILUVW FOXE DQG FKRRVLQJ WKH ILUVW ER\ ZKR LV
ILUVW ER\ LQ WKH VHFRQG FOXE DQG FRQLQXLQJ LQ W
ER\ V LQ WKH VHFRQG FOXE 1RWH WKDW HDFK ER\ LQ V
ER\ LQ WKH ILUVW FOXE ZKR LV WDOOHU WKDQ KLP RW
1RZ IURP DPRQJ WKH ER\ ZKR DUH QRW LQ HLWKHU R
FOXE VWDUWLQJ ZLWK WKH ILUVW ER\ QRW LQ WKH ILU
WKDQ Q ER\ V LQ WKH WKLV FOXE DQG HDFK RI WKHVH
ZKR LV WDOOHU WKHQ KLP :H FRQLQXH LQ WKLV ZD\
PRUH WKDQ Q PHPEHUV DQG HDFK PHPEHU RI HDFK FO
FOXE

7KHU DUH DWPRUW LQ WKHVH Q FOXEV VR QRZ OHW X
DQ\ RI WKHVH FOXEV +H PXVW IROORZ D WDOOHU ER\
WK

