THE 2007 2008 KENNESAW STATE UNIVERSITY HIGH SCHOOL MATHEMATICS COMPETITION

PART I ±MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend, or write stray marks on either side of the answer sheet. Each correct answer is worth 6 points. Two points are given if no box, or more than one box, is marked. Zero points are given for an incorrect answer. Note that wild guessing is apt to lower your score. When the exam is over, give your answer sheet to your proctor. You may keep your copy of the questions.

NO CALCULATORS

90 MINUTES

1. How many three digit positive integers are there such that the sum of the digits is a multiple of 7, the first two digits add to 12, and the number contains a repeated digit?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

2. Which of these numbers is the average (mean) of the other four?

   (A) 27 (B) 36 (C) 25 (D) 29 (E) 28

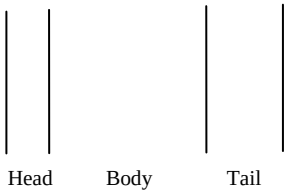
3. If $\frac{a \quad 2b}{a \quad 2b} \quad 3$, what is the value of $\frac{a \quad 3b}{a \quad 3b}$?

   (A) 7 (B) 6 (C) 4 (D) 2 (E) None of these

4. The sum of the lengths of three of the four sides of a rectangle is 2007. The sum of the length of the fourth side and the length of a diagonal of the rectangle is also 2007. What is the ratio of the length of the longer side to the length of the shorter side of this rectangle.

   (A) $\sqrt{2}:1$ (B) $\sqrt{3}:1$ (C) 2:1 (D) 3:1 (E) 4:1

5. A fish had a tail as long as its head plus a quarter the length of its body. Its body was three-fourths of its total length. If its head was 4 centimeters long, what was the entire length of the fish?



   (A) 100 cm (B) 120 cm (C) 128 cm (D) 132 cm (E) 136 cm

Continued on back

6. What is the value of $\log\frac{1}{2}\ \ \log\frac{2}{3}\ \ \log\frac{3}{4}\ \ \log\frac{4}{5}\ \ ...\ \ \log\frac{99}{100}$?

(A) 0 (B) 1 (C) 2 (D) -1 (E) -2

7. Two people take turns rolling a die. What is the probability that the second person will roll a 1 before the first person rolls a 6?

(A) $\frac{1}{2}$ (B) $\frac{5}{11}$ (C) $\frac{7}{12}$ (D) $\frac{13}{36}$ (E)

8.

13. If x and n are positive integers such that $x^2 \quad 615 \quad 2^{2n}$, what is the value of x + n?

14. Let d represent the length of the diagonal of a cube. Which of the following represents the surface area of the cube?

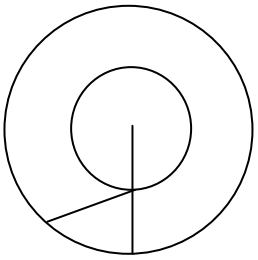
(A) $\sqrt{}$ (B) $d^2\sqrt{3}$ (C) $\frac{3}{2}d^2$ (D) $2d^2$ (E) $3d^2$

15. Find the sum of all values of x which satisfy: ———— ———— ————.

(A) 29 (B) 38 (C) 45 (D) 69 (E) None of these

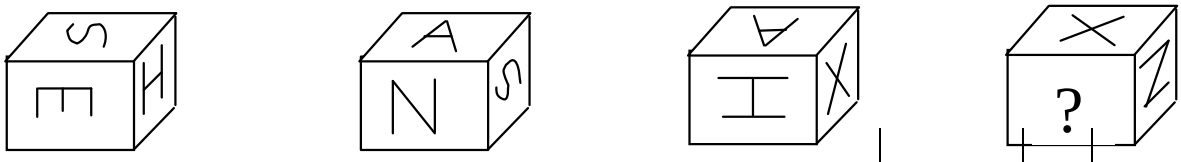
16. The number 2007 has N factors (including itself and 1). Compute the number of two-digit positive integers which have exactly N factors.

20. Two concentric circles are shown. The radius of the inner circle is 3, and the distance between the circles is 3. A line segment of length 4 has its endpoints on both circles. Compute the distance from point A to point B .



(A) $\sqrt{7}$ (B) $\sqrt{14}$ (C) $\sqrt{15}$ (D) $\sqrt{19}$ (E) 5

21. Below are four different views of the same toy alphabet block. Which of the following should appear on the blank (where the **?** is).



(A) (B) (C) (D) (E)

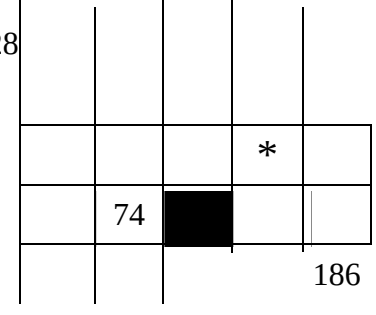
22. Let $P(x) = x^4 \quad ax^3 \quad bx^2 \quad cx \quad d$. If P(1) = 10, P(2) = 20, and P(3) = 30, compute the value of P(10) + P(-6).

(A) 4896 (B) 5240 (C) 6064 (D) 7816 (E) 8104

23. Of the animals entered in a dog show, the number of poodles is at least one-fifth of the number of beagles and at most one-sixth the number of collies. The number of dogs which are poodles or beagles is at least 23. What is the minimum number of collies entered in this show?

(A) 20 (B) 22 (C) 24 (D) 26 (E) 28

24. It is possible to place positive integers into the twenty-one vacant squares of the 5x5 square shown at the right, so that the numbers in each row and each column form arithmetic sequences. What number must occupy the square marked by the asterisk ( ).



(A) 118 (B) 126 (C) 134 (D) 142 (E) 150

25. One vertex of an equilateral triangle lies on the point with coordinates (1, 4). The other two vertices lie on the line whose equation is $y = 3x \pm 4$, at the points (x_1, y_1) and (x_2, y_2). Compute the sum $y_1 + y_2$.

(A) 7 (B) 7.5 (C) 8 (D) 8.5 (E) None of these

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Part I ± Solutions:

1. E Let a, b, and c be the digits. The sum of the digits must be 14 or 21. Thus $a + b + c = 14$ or $a + b + c = 21$. Since $a + b = 12$, $c = 2$ or $c = 9$. If $c = 2$, then a and b must both be 6. If $c = 9$, then a and b could both be six, or one could be 9 and the other 3. Hence the possibilities are 662, 669, 939, 993, for a total of four.

2. D Although the problem can be done by trial and error using the choices, note that if one of the numbers is the mean of the other four, it is the mean of all 5.
$\frac{27 \quad 36 \quad 25 \quad 29 \quad 28}{5}$ 29.

3. A $\frac{a \quad 2b}{a \quad 2b}$ 3 Ÿ a = 4b. Substituting $\frac{a \quad 3b}{a \quad 3b} \quad \frac{4b \quad 3b}{4b \quad 3b} \quad \frac{7b}{b}$ 7.

4. B We are given 2b + a = 2007 and a + d = 2007. Subtracting the second equation from the first, 2b = d. Substituting into $a^2 \quad b^2 \quad d^2$, we get $a^2 \quad b^2 \quad 4b^2$, from which we obtain $a^2 \quad 3b^2$ and $\frac{a}{b} \quad \frac{\sqrt{3}}{1}$

5. C

9. C Since B‰C = {1,2,3,4} and Bˆ C = {1,2}, there ar

21. B

25. A Let T be the measure of the angle the given line makes with the positive x-axis.